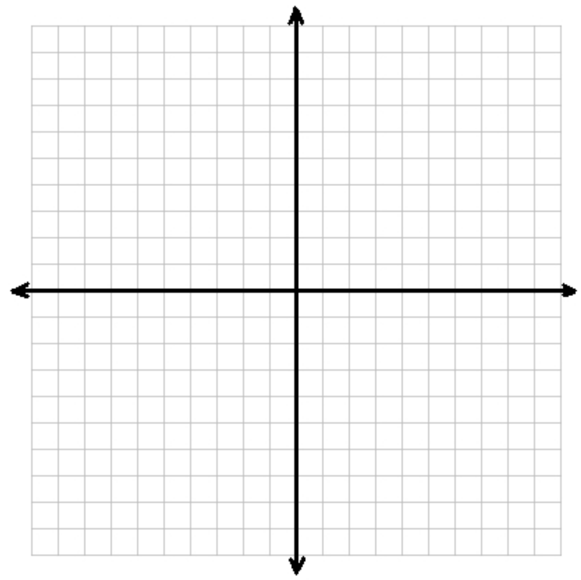
MATH 1650: SECTION 1.3: ABSOLUTE VALUE FUNCTIONS

EXAMPLE: Graph $f(x) = |x|$ set of axes below.

x	$f(x) = \lvert x \rvert$	$(x, f(x))$



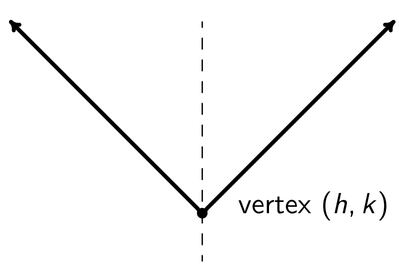
We describe the graph of $f(x) = |x|$ as a '$\vee$' shape with **vertex** at $(0, 0)$ opening **upwards** with **slopes** ± 1.

EXAMPLE: Graph each of the following functions using a graphing utility and record the information requested.

- $f(x) = |x - 3| + 2$
  - vertex:
  - opens: up or down
  - slopes of lines:
- $f(x) = -2|x + 1| + 4$
  - vertex:
  - opens: up or down
  - slopes of lines:
- $f(x) = \frac{1}{2}|x - 1| - 2$
  - vertex:
  - opens: up or down
  - slopes of lines:

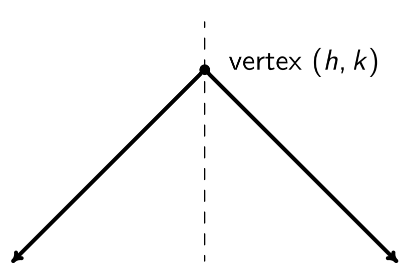
IN GENERAL: GRAPHS OF BASIC ABSOLUTE VALUE FUNCTIONS

The graph of $f(x) = a|x - h| + k$ is a '$\vee$' shape with vertex (h, k) with lines with slopes $\pm a$:



Axis of Symmetry, $x = h$

$a > 0$; Minimum is k, no maximum



Axis of Symmetry, $x = h$

$a < 0$; Maximum is k, no minimum

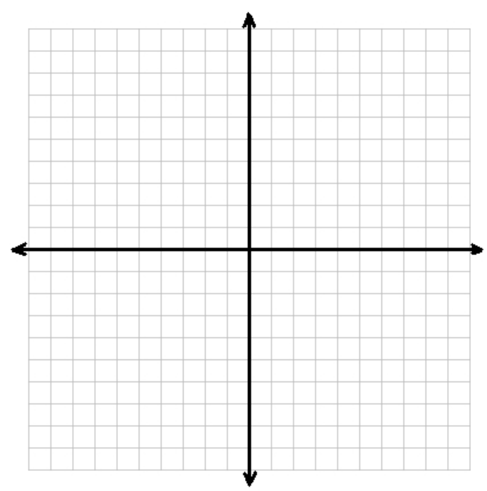
EXAMPLE: Consider: $f(x) = 2|x+3| - 1$.

- Find the vertex without using a graphing utility:

- Find $f(0)$ and use this to find the y-intercept:

- Solve $f(x) = 0$ and use this to find the x-intercepts:

- Graph $f(x)$ **by hand** - that is, **without** using a graphing utility. Plot the vertex and intercepts.



- Using interval notation, state:

  the **domain** of f: the **range** of f:

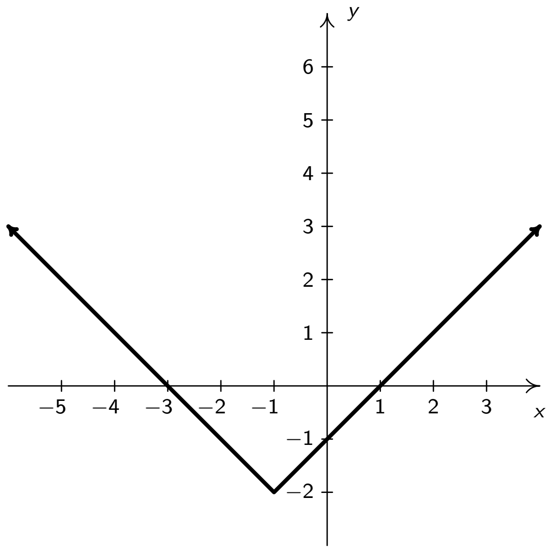
- state the:

  the **maximum** of f, if there is one: the **minimum** of f, if there is one:

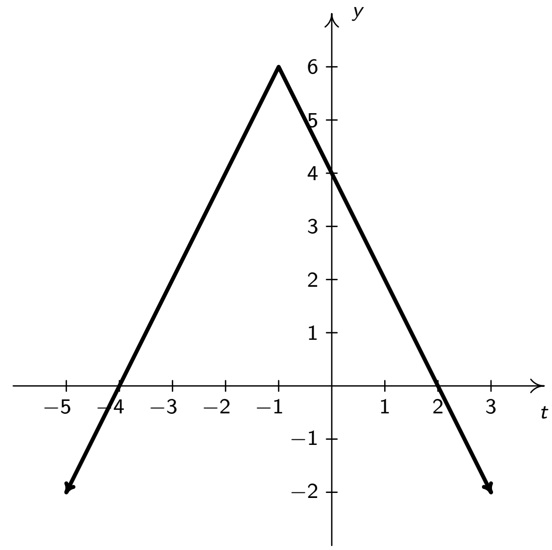
- State the intervals (if any) over which f is:

  increasing: **decreasing**: **constant**:

EXAMPLE: Find a possible formula for each function whose graph appears below:



$y = f(x)$



$y = G(t)$

$f(x) =$ ____________ $G(t) =$ ____________

EXAMPLE: Consider $f(x) = |-2x+5|+4$. While not in the form $f(x) = a|x-h|+k$, we can rewrite $f(x)$ as:

$$f(x) = |-2x+5|+4 = \left|(-2)\left(x-\frac{5}{2}\right)\right| + 4 = 2\left|x-\frac{5}{2}\right| + 4$$

If written $f(x) = 2\left|x-\frac{5}{2}\right| + 4$, we quickly identify the vertex as $\left(\frac{5}{2}, 4\right)$ and graph this function by hand.

EXAMPLE: Rewrite each of the functions below in the form $f(x) = a|x-h|+k$. Identify the vertex.

- $f(x) = |2x+3| - 4$
- vertex:
- $f(x) = |5-x| + 1$
- vertex:
- $f(x) = \dfrac{|x-3|+1}{2}$
- vertex:
- $f(x) = \left|\dfrac{5-2x}{3}\right| - 4$
- vertex:

DEFINING THE ABSOLUTE VALUE AS A PIECEWISE-DEFINED FUNCTION:

EXAMPLE: Consider $f(x) = |x-3| - x + 2$. $f(x)$ **cannot** be written in the form $f(x) = a|x-h| + k$. Why?

Another way to describe the absolute value function is that it 'makes negative numbers positive.' We have:

PIECEWISE DEFINITION: $|x| = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$

NOTE: '$-x$' is most correctly read as 'the opposite of x.'

EXAMPLE: Use the piecewise definition of absolute value to find: $|5|$, $|0|$, and $|-5|$.

- Since $5 \geq 0$, to find $|5|$ we use the rule $|x| = x$, so $|5| = 5$.
- Since $0 \geq 0$, to find $|0|$ we use the rule $|x| = x$, so $|0| = 0$.
- Since $-5 < 0$, to find $|-5|$ we use the rule $|x| = -x$, so $|-5| = -(-5) = 5$.

EXAMPLE: Use the piecewise definition of $|x|$ to rewrite $f(x) = |x-3| - x + 2$ as a piecewise-defined function.

Since $|x| = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$, we replace '$x$' with '$(x-3)$' to get a formula for $|x-3|$:

$$|x-3| = \begin{cases} -(x-3), & \text{if } (x-3) < 0 \\ (x-3), & \text{if } (x-3) \geq 0 \end{cases},$$

We simplify: $-(x-3) = -x+3$ and $(x-3) = x-3$. We solve $(x-3) < 0$ as $x < 3$ and $(x-3) \geq 0$ as $x \geq 3$.

$$|x-3| = \begin{cases} -x+3, & \text{if } x < 3 \\ x-3, & \text{if } x \geq 3 \end{cases}.$$

Next, we use this piecewise formula for $|x-3|$ to break $f(x) = |x-3| - x + 2$ into two pieces.

- If $x < 3$, $|x-3| = -x+3$, so $f(x) = |x-3| - x + 2 = -x + 3 - x + 2 = -2x + 5$.
- If $x \geq 3$, $|x-3| = x-3$, so $f(x) = |x-3| - x + 2 = x - 3 - x + 2 = -1$.

Putting this all together, we get:

$$f(x) = \begin{cases} -2x+5, & \text{if } x < 3 \\ -1, & \text{if } x \geq 3 \end{cases}$$

Rewriting $f(x)$ as a piecewise-defined function in this way allows us to graph $f(x)$ **by hand**.

EXAMPLE: Let $f(x) = |2x - 3| + x - 4$. Write $f(x)$ as a piecewise-defined function.

1. Write $|2x - 3|$ as a piecewise-defined function.

   Since $|x| = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$, we replace '$x$' with '$(2x-3)$' to get a formula for $|2x - 3|$:

   $$|2x - 3| = \begin{cases} -(2x-3), & \text{if } (2x-3) < 0 \\ (2x-3), & \text{if } (2x-3) \geq 0 \end{cases},$$

   - Simplify: $-(2x - 3) =$

   - Simplify: $(2x - 3) =$

   - Solve: $(2x - 3) < 0$:

   - Solve: $(2x - 3) \geq 0$:

   Hence,

   $$|2x - 3| = \begin{cases} \underline{\hspace{3cm}}, & \text{if } \underline{\hspace{3cm}} \\ \\ \underline{\hspace{3cm}}, & \text{if } \underline{\hspace{3cm}} \end{cases}$$

2. Next, we use the pieces from $|2x - 3|$ to break $f(x)$ into pieces:

   - If $x < \dfrac{3}{2}$, $|2x-3| = -2x+3$, so $f(x) = |2x-3|+x-4 = \underline{\hspace{3cm}} + x - 4 = \underline{\hspace{3cm}}$

   - If $x \geq \dfrac{3}{2}$, $|2x-3| = 2x-3$, so $f(x) = |2x-3|+x-4 = \underline{\hspace{3cm}} + x - 4 = \underline{\hspace{3cm}}$

   Putting it all together, we get $f(x) = \begin{cases} \underline{\hspace{3cm}}, & \text{if } \underline{\hspace{3cm}} \\ \\ \underline{\hspace{3cm}}, & \text{if } \underline{\hspace{3cm}} \end{cases}$

HOMEWORK: Section 1.3: 1 - 37 odd.